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# Determining the CP-eigenvalues of the Neutral Higgs Bosons of the Minimal Supersymmetric Model in $\gamma\gamma$ Collisions

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## Abstract

We determine the optimal laser and  $e^+/e^-$  energies and polarizations for *directly* determining the CP eigenvalue of each of the neutral Higgs bosons of the Minimal Supersymmetric Model via measurement of transverse polarization cross section asymmetry in back-scattered laser photon collisions. Approximate statistical significances are computed for the measurement of the CP eigenvalue as a function of Higgs mass and other parameters of the model.

## 1. Introduction

Supersymmetric models are of considerable interest as a possibility for extending the Standard Model (SM).<sup>[1]</sup> A supersymmetric extension of the SM must contain at least two Higgs doublets. The minimal supersymmetric model (MSSM) is defined by having precisely two Higgs doublets. The resulting physical Higgs sector contains two charged Higgs bosons ( $H^\pm$ ), two CP-even Higgs bosons ( $h^0, H^0$  with  $m_{h^0} < m_{H^0}$ ), and one CP-odd Higgs boson ( $A^0$ ). (Note that CP is automatically conserved in the MSSM Higgs sector, by virtue of the SUSY constraints.) At tree-level, the MSSM Higgs sector is entirely determined by just two parameters, which are normally chosen to be  $\tan\beta \equiv v_2/v_1$  (the ratio of the vacuum expectation values of the neutral members of the Higgs doublets that couple to up- and down-type quarks, respectively) and  $m_{A^0}$ . One-loop radiative corrections are, however, important.<sup>[2]</sup> They depend most crucially upon the top quark mass ( $m_t$ ) and the stop squark mass ( $m_{\tilde{t}}$ ). However, other parameters enter as well. These include the soft supersymmetry breaking parameters,  $A_t, A_b$ , and the  $\mu$  parameter characterizing Higgs superfield mixing.

Recently, attention has been given to the possibility of forming Higgs bosons via polarized photon collisions.<sup>[3,4]</sup> Intense beams of polarized photons can be produced by back-scattering polarized laser beams off of polarized electron and positron beams at a TeV-scale linear  $e^+e^-$  collider.<sup>[5,6]</sup> In previous work,<sup>[3]</sup> it has been established that the neutral MSSM Higgs bosons can indeed be detected in  $\gamma\gamma$  collisions over much of parameter space. In fact, since each can be produced singly by direct  $\gamma\gamma$  collisions, whereas they are only detectable in  $e^+e^-$  collisions in the pair production mode,  $e^+e^- \rightarrow A^0 H^0$ , photon-photon colliders can even have a larger mass reach in the case of the heavier  $A^0$  and  $H^0$  than direct  $e^+e^-$  collisions.<sup>[3]</sup>

Once Higgs bosons are observed in either  $e^+e^-$  collisions or  $\gamma\gamma$  collisions, it will be crucial to check their detailed properties. Since one of the most basic features of the MSSM Higgs sector is the prediction that *pure* eigenstates with CP=+ and CP=- must both be present, it will be very important to make a direct determination of the CP of each Higgs boson that

is detected. Direct measurement of the CP-properties of the three neutral Higgs bosons will, however, be challenging at any future collider.

A  $\gamma\gamma$ -collider allows a study of the CP-properties of the Higgs sector by virtue of the different couplings for two photons to CP-even vs. CP-odd Higgs bosons:

$$\mathcal{M}[\gamma\gamma \rightarrow h(CP = ++)] \propto \vec{\epsilon} \cdot \tilde{\vec{\epsilon}}, \quad \mathcal{M}[\gamma\gamma \rightarrow h(CP = +-)] \propto (\vec{\epsilon} \times \tilde{\vec{\epsilon}})_z, \quad (1)$$

where  $\vec{\epsilon}$  and  $\tilde{\vec{\epsilon}}$  are the polarizations of the two colliding photons in the  $\gamma\gamma$  CM. This was first exploited in Ref. [7] to show that a Higgs boson of mixed CP character could be distinguished from a pure CP eigenstate by the fact that a certain asymmetry between production cross sections initiated by colliding photons of different helicities was non-zero. In contrast, it was noted in Refs. [7] and [8] (see also Ref. [10]) and it is evident from Eq. (1) that determination of the CP of a pure eigenstate requires transversely polarized colliding photons and measurement of the transverse polarization asymmetry between event rates for parallel linear polarizations versus perpendicular linear polarizations of the colliding photons. As discussed in Refs. [7,8-10], and detailed here, the photons coming from the Compton back-scattering process can at best be only partially polarized in the transverse direction. We will discuss means for optimizing the observability of the transverse polarization asymmetry by adjusting machine energy, electron and positron polarizations, laser energy and laser polarizations.<sup>[11]</sup>

For near optimal choices, we compute the degree to which the CP-eigenvalues of each of the three neutral Higgs bosons will be measurable; results are given as a function of the fundamental parameters which determine the MSSM Higgs boson masses and decays. For each of the Higgs bosons we survey the most important final state channels for this determination. We find that the chances for measurement of the CP-eigenvalue of the  $h^0$  are generally good; prospects in the case of the  $H^0$  and  $A^0$  are much more dependent upon model parameters, but can also be good. Overall, this technique should prove very valuable, especially if it is kept in mind that CP determination will only be attempted after the Higgs boson(s) have already been observed, *i.e.* as part of a second generation, hopefully high luminosity experiment.

## 2. Procedure and Results

The event rate for  $\gamma\gamma \rightarrow h \rightarrow X$  where  $h$  is the neutral Higgs boson of interest and  $X$  is a particular final state can be written

$$\begin{aligned} \frac{dN}{dt}(\gamma\gamma \rightarrow h \rightarrow X) &= 8\pi \frac{BR(h \rightarrow X)}{E_{e^+e^-} m_h^2} \tan^{-1} \frac{\Gamma_{res}}{\Gamma_h} \Gamma(h \rightarrow \gamma\gamma) \frac{d\mathcal{L}_{\gamma\gamma}}{dy}|_{y_h} \\ &\times \left\{ 1 + \langle \xi_2 \tilde{\xi}_2 \rangle + (\langle \xi_3 \tilde{\xi}_3 \rangle - \langle \xi_1 \tilde{\xi}_1 \rangle) \mathcal{A}_3 \right\} \\ &\equiv \frac{d}{dt} \left( N_{1+\langle \xi_2 \tilde{\xi}_2 \rangle} + \cos(2\kappa) N_{\mathcal{A}_3} \right). \end{aligned} \quad (2)$$

In Eq. (2),  $BR(h \rightarrow X)$  is the  $h \rightarrow X$  branching ratio,  $\Gamma(h \rightarrow \gamma\gamma)$  is the  $h \rightarrow \gamma\gamma$  width,  $\Gamma_h$  is the total Higgs width,  $m_h$  is the Higgs mass,  $E_{e^+e^-}$  is the center-of-momentum (CM) energy

of the  $e^+e^-$  collider, and we have given the result appropriate for a CP-conserving Higgs sector. The asymmetry  $\mathcal{A}_3$  is that defined in Ref. [7], and will be given explicitly shortly. For the  $Q\bar{Q}$  channels, which have a substantial background, we take  $\Gamma_{res}$  to be either the best achievable experimental resolution,  $\Gamma_{exp}$ , or  $\Gamma_h$ , whichever is larger. This minimizes the background level. The other channels considered are largely background free, and  $\Gamma_{res}$  is taken to be large (implying  $\tan^{-1} \sim \pi/2$ ). In Eq. (2),  $\frac{d\mathcal{L}_{\gamma\gamma}}{dy}|_{y_h}$  is the  $\gamma\gamma$ -luminosity averaged over collisions for fixed  $e^+e^-$  CM energy, evaluated at  $y_h \equiv E_{\gamma\gamma}/E_{e^+e^-} = m_h/E_{e^+e^-}$ ;  $\kappa$  is the angle between the directions of maximum linear polarization of the two laser beams, assumed to be approaching nearly head-on; and  $\xi_i$  and  $\tilde{\xi}_j$  ( $i, j = 1, 2, 3$ ) are the Stokes parameters which specify the polarization state of the photons  $\gamma$  and  $\tilde{\gamma}$ , respectively. For more details on the notation see Ref. [6]. The Stokes parameters  $\xi_{1,3}$  and  $\tilde{\xi}_{1,3}$  depend on  $\kappa$ ; we have extracted the form of this dependence explicitly in defining  $N_{\mathcal{A}_3}$ . In our notation, the brackets  $\langle \dots \rangle$  denote averaging over collisions at fixed  $y_h$  (which differs from the meaning of  $\langle \dots \rangle$  in Ref. [5]).

The asymmetry  $\mathcal{A}_3$  can be written in terms of either linear polarization or helicity amplitudes for Higgs production in  $\gamma\gamma$  collisions:

$$\mathcal{A}_3 \equiv \frac{|\mathcal{M}_{\parallel}|^2 - |\mathcal{M}_{\perp}|^2}{|\mathcal{M}_{\parallel}|^2 + |\mathcal{M}_{\perp}|^2} = \frac{2\text{Re}(\mathcal{M}_{--}^* \mathcal{M}_{++})}{|\mathcal{M}_{++}|^2 + |\mathcal{M}_{--}|^2}, \quad (3)$$

where the  $\mathcal{M}$ 's are the amplitudes for photons of the indicated relative linear polarizations or helicities. As already noted in association with Eq. (1),  $\mathcal{A}_3$  is determined by the CP-eigenvalue of the Higgs boson produced;  $\mathcal{A}_3 = +1(-1)$  for CP-even (-odd) scalars.<sup>[7]</sup> We wish to isolate  $\mathcal{A}_3$  in the event rate in order to determine the CP of an observed  $h$ . This is made possible by the  $\kappa$  dependence appearing in Eq. (2) — the  $\mathcal{A}_3$  term depends upon  $\cos(2\kappa)$ , whereas the  $1 + \langle \xi_2 \tilde{\xi}_2 \rangle$  term is independent of  $\kappa$ .  $\mathcal{A}_3$  is thus isolated by taking the difference between event rates at  $\kappa = 0$  and  $\kappa = \frac{\pi}{2}$ , *i.e.* parallel vs. perpendicular linear polarizations for the initial laser beams.

Regarding the Higgs widths, we note that for the MSSM Higgs bosons,  $\Gamma_h$  is typically much smaller than expected experimental resolutions, although for large values of  $\tan\beta$  the  $b\bar{b}$  coupling is enhanced and the associated decay width can be significant for the  $A^0$  and either the  $h^0$  (if  $m_{A^0}$  is small) or the  $H^0$  (if  $m_{A^0}$  is large). Also, when decay into  $t\bar{t}$  is kinematically allowed the  $A^0$  and  $H^0$  widths can be significant even if  $\tan\beta$  is not large. Nonetheless, these widths are never much larger than 5 to 10 GeV for the parameter ranges considered here; our choice of  $\Gamma_{exp} = 15$  GeV is generally larger. However,  $\tan^{-1}[\Gamma_{exp}/\Gamma_h]$  in Eq. (2) should not be approximated by simply using the  $\pi/2$  limit.

There are two sources of background to be considered. One is due to the  $1 + \langle \xi_2 \tilde{\xi}_2 \rangle$  term for the Higgs itself which induces an error in both of the accumulated event numbers,  $N|_{\kappa=0}$  and  $N|_{\kappa=\frac{\pi}{2}}$ . The second derives from continuum background(s) in the channel(s) in which the  $h$  is detected (which are  $\kappa$  independent assuming a sum over all events subject only to rapidity and/or polar angle cuts). One measure of the significance of the  $\mathcal{A}_3$  signal is the

number of standard deviations by which  $(N|_{\kappa=0} - N|_{\kappa=\frac{\pi}{2}})$  exceeds the expected uncertainty of the combined background distribution:

$$N_{SD}(\mathcal{A}_3) = \frac{\sqrt{2}N_{\mathcal{A}_3}}{\sqrt{N_{1+\langle\xi_2\tilde{\xi}_2\rangle} + N_{cont}}}, \quad (4)$$

where  $N_{\mathcal{A}_3}$  and  $N_{1+\langle\xi_2\tilde{\xi}_2\rangle}$  are defined by Eq. (2), and  $N_{cont}$  is the event number for the continuum background. We employ  $N_{SD}(\mathcal{A}_3)$  to determine the range of MSSM Higgs sector parameters for which the CP-eigenvalue of an MSSM  $h$  might be measurable. We shall evaluate  $N_{SD}(\mathcal{A}_3)$  for a yearly integrated  $\gamma\gamma$  luminosity of  $20 \text{ fb}^{-1}$  (a frequently employed canonical value for the first few years of operation of a back-scattered laser beam facility).

A crucial issue is the best means for maximizing  $N_{SD}(\mathcal{A}_3)$ .  $N_{\mathcal{A}_3}$  will be largest when the transverse polarizations of the colliding photons are as big as possible. For a given machine configuration this means employing laser beams with maximal transverse polarizations; we assume that essentially perfect transverse polarization will be possible:  $P_T(\gamma_1) = P_T(\gamma_2) = 1$ . (Our notation is that  $\gamma_1$  collides with the  $e^-$  and gives rise to back-scattered photon  $\gamma$ , while  $\gamma_2$  collides with the  $e^+$  producing  $\tilde{\gamma}$ . The associated Stokes parameters are  $\xi_i$  and  $\tilde{\xi}_i$ , respectively.) For  $P_T(\gamma_1) = 1$  the value of  $\xi_3$ , for laser polarization orientation such that  $\xi_1 = 0$ , is given by

$$\xi_3 = \frac{2r^2}{(1-z)^{-1} + (1-z) - 4r(1-r)}, \quad (5)$$

where  $r = zx^{-1}/(1-z)$  with  $z = E_\gamma/E_{e^-}$ . A similar result applies for  $\tilde{\xi}_3$ . The parameter  $x$  in the algebraic form for  $r$  is determined by the  $e^+e^-$  CM energy,  $E_{e^+e^-}$ , and the photon energy of the laser,  $\omega_0$ :  $x = \frac{2E_{e^+e^-}\omega_0}{m_e^2}$ . We assume in the following that both lasers are operated at the same  $x$  value;  $x$  must be less than 4.83 in order that the back-scattering process be below the pair production threshold. The maximum value of  $z$  occurs for  $r = 1$ , *i.e.*  $z_{max} = x/(1+x)$ , at which point  $\xi_3$  reaches its largest value,  $\xi_3(z_{max}) = 2(1+x)/[1+(1+x)^2]$ . This obviously suggests that choosing small values of  $x$ , and operating at a machine energy that demands  $z$  values near  $z_{max}$  for a given (known) Higgs mass, will yield optimal results. However, choosing a machine energy (assuming for the moment that adequate machine energy is available) such that  $y_h = x/(1+x)$ , which would force  $z = \tilde{z} = x/(1+x)$ , is too extreme since the folded luminosity function would be zero. We return to this point below. First, we also note that from the form of  $\xi_3(z_{max})$  as a function of  $x$  it is clear that there is diminishing return in lowering  $x$  much below 1.

In this paper, we shall assume a minimum possible value for  $x$  of  $x_{min} = 0.5$ , but that, when necessary, the lasers can be operated at higher values of  $x$  (up to the  $x \sim 4.8$  pair threshold point). Assuming for the moment no limitation on machine energy, we have searched for the choice of  $y_h$  that maximizes  $N_{SD}(\mathcal{A}_3)$  for a given value of  $x_{min}$ . The results

are most easily summarized in the form

$$y_h^{opt}(x_{min}) = \alpha(x_{min}) \frac{x_{min}}{(1 + x_{min})}. \quad (6)$$

The values of  $\alpha(x_{min})$  as we vary  $x_{min}$  from 4.8 down to .2 in steps of .1 are given in Table 1. We see that the optimal value of  $y_h$  is never much below the kinematic maximum. In our results we shall always employ a machine energy given by  $E_{e^+e^-}^{opt} = m_h/y_h^{opt}(x_{min})$  whenever this energy does not exceed the maximum machine energy assumed available, denoted by  $E_{e^+e^-}^{mach}$ . For our particular choice of  $x_{min} = 0.5$ ,  $y_h^{opt}(x_{min}) \sim 0.299$ . Using this, the defining formula for  $x$  with  $x = x_{min} = 0.5$ , and  $E_{e^+e^-}$  set equal to  $E_{e^+e^-}^{opt}$ , we find that the laser energy  $\omega_0$  is given in terms of the Higgs mass (in GeV) by:

$$\omega_0 \sim \frac{19.5 \text{ eV}}{m_h(\text{GeV})}, \quad (7)$$

*i.e.* lasers with energies in the fractional eV range would generally be necessary to operate at the optimal point.

Table 1: We tabulate the values of  $\alpha(x)$ , Eq. (6), for  $x$  ranging from 4.8 down to .2 in steps of .1.

0.966	0.966	0.966	0.964	0.964	0.964	0.962	0.962	0.962	0.960	0.960	0.960
0.958	0.958	0.958	0.956	0.956	0.954	0.954	0.952	0.952	0.950	0.949	0.948
0.947	0.946	0.944	0.942	0.940	0.938	0.937	0.936	0.934	0.932	0.928	0.926
0.924	0.920	0.918	0.914	0.910	0.906	0.902	0.898	0.892	0.888	0.884	—

For larger  $m_h$  values, it is frequently the case that  $E_{e^+e^-}^{opt} > E_{e^+e^-}^{mach}$ . In this situation, we have explicitly searched for the compromise values of  $x$  and  $y_h$  that maximize  $N_{SD}(\mathcal{A}_3)$ , subject to the restrictions  $E_{e^+e^-} \leq E_{e^+e^-}^{mach}$ ,  $x_{min} \leq x \leq 4.8$ . For all such cases considered we have found that it is always best to employ  $E_{e^+e^-} = E_{e^+e^-}^{mach}$ . The best choice of  $x$  is situation dependent, but does tend to be fairly near the minimum possible choice of  $x = y_h^{mach}/(1 - y_h^{mach})$ , where  $y_h^{mach} = m_h/E_{e^+e^-}^{mach}$ .

Two additional notes are useful. First, the optimal choices outlined above are independent of the particular Higgs boson decay channel being considered. Second, the best choice for  $y_h$  is always fairly close to its kinematical limit. This means that the  $\gamma\gamma$  subprocess energy is such that  $Q\bar{Q}$  backgrounds arising from “resolved-photons”<sup>[12]</sup> will be very suppressed compared to the direct  $\gamma\gamma \rightarrow Q\bar{Q}$  subprocess continuum backgrounds that we shall consider. This is simply due to the fact that if an incoming photon radiates a secondary particle which participates in  $Q\bar{Q}$  production, there is a very low probability that the  $Q\bar{Q}$  pair will have invariant mass large enough to be confused with a signal at the (known) Higgs mass.

Finally, a small amount of optimization of  $N_{SD}(\mathcal{A}_3)$  is possible by choosing  $e^-$  and  $e^+$  helicities such that  $1 + \langle \xi_2 \tilde{\xi}_2 \rangle$  is as small as possible. (This turns out to be appropriate even in the  $b\bar{b}$  and  $t\bar{t}$  channels where the continuum backgrounds increase with decreasing  $1 + \langle \xi_2 \tilde{\xi}_2 \rangle$ .) Our results are given for  $\langle \lambda(e^-) \rangle = -\langle \lambda(e^+) \rangle = 0.45$ . However the quoted  $N_{SD}(\mathcal{A}_3)$  values are decreased by at most 10% if the  $e^-$  and  $e^+$  beams are unpolarized.

We now turn to quantitative results. In the case of the  $Q\bar{Q}$  channels we employ an experimental mass resolution in the final state of  $\Gamma_{exp} = 15$  GeV. We give results assuming the rumored CDF top quark mass value of 175 GeV. For the  $Q\bar{Q}$  final states, we impose an angular cut in the  $\gamma\gamma$  CM frame of  $|\cos\theta| < 0.8$ . We then compute  $N_{SD}(\mathcal{A}_3)$  as a function of the MSSM parameter  $m_{A^0}$ , varying  $m_{A^0}$  between 40 and 800 GeV, for the two representative values of  $\tan\beta = 2, 20$ . Radiative corrections for the  $h^0$  and  $H^0$  masses are computed neglecting squark mixing. Initially, we shall employ  $m_{\tilde{t}} = 1$  TeV. Alterations in our results for lower values of the squark masses will be summarized towards the end of the paper.

Below, we summarize results for the decay channels with highest  $N_{SD}(\mathcal{A}_3)$ , using an integrated photon-photon luminosity of  $L = 20 \text{ fb}^{-1}$ . While such an integrated luminosity will not always turn out to be adequate for the determination of  $\mathcal{A}_3$ , it is to be hoped that higher luminosities could eventually be accumulated. Since both signal and background scale as  $L$ ,  $N_{SD}(\mathcal{A}_3)$  scales as  $\sqrt{L}$ .

Before beginning our survey, it is useful to recall some basic features of the decays of the Higgs bosons (see Refs. [1] and [8] for more detailed discussions) in order to better understand why particular channels yield the best  $N_{SD}(\mathcal{A}_3)$  values. First, we emphasize that we have considered in this work only those channels that contain standard model particles. Once more is known experimentally about the supersymmetric spectrum (for which an  $e^+e^-$  collider should be an excellent machine) the SUSY decay modes for the Higgs bosons can be computed, and a reassessment of all channels, including SUSY channels, performed. We shall discuss the modifications in SM channels for a sample case in which SUSY modes are allowed. A study of the possibilities for measuring  $\mathcal{A}_3$  using SUSY decay modes is beyond the scope of this study. Possible problems include invisible decay modes such as  $h \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ , and, more generally, the presence of missing energy that could worsen the experimental resolution in a typical SUSY decay channel, making it more difficult to reject background events. For our basic scenario we shall adopt a universal soft SUSY-breaking squark mass of 1 TeV, and the masses of charginos and neutralinos will be set to very large values by the soft-SUSY breaking parameter choices  $M = -\mu = 1$  TeV. Squark mixing will be neglected.

In the absence of SUSY modes, decays of the  $h^0$  are dominated by the  $b\bar{b}$  channel, independent of the value of  $\tan\beta$ . ( $m_{h^0}$  is always smaller than  $2m_t$ .) In contrast, the decays of the  $A^0$  and  $H^0$  change substantially as  $\tan\beta$  is increased. For small to moderate  $\tan\beta$ , the primary  $A^0$  decays are to  $b\bar{b}$  so long as  $m_{A^0} < m_Z + m_{h^0}$ . However, once the  $A^0$  mass is above the  $Zh^0$  threshold,  $A^0 \rightarrow Zh^0$  decays tend to dominate, although the  $b\bar{b}$  channel still remains significant. For small to moderate  $\tan\beta$ , the  $H^0$  decays primarily to  $b\bar{b}$  for  $m_{H^0} < 2m_{h^0}$ , while  $H^0 \rightarrow h^0 h^0$  decays are dominant once  $m_{H^0} > 2m_{h^0}$ . Of course, for  $m_{A^0}, m_{H^0}$  above  $2m_t$ ,  $A^0, H^0 \rightarrow t\bar{t}$  decays dominate for small to moderate  $\tan\beta$ . At large  $\tan\beta$ , the couplings of the  $A^0$  and  $H^0$  to  $b\bar{b}$  and  $\tau^+\tau^-$  become highly enhanced, and these

Figure 1:  $N_{SD}(\mathcal{A}_3)$  for the  $h^0$  is displayed as a function of  $m_{h^0}$  for  $\tan\beta = 2$  and 20 for the  $b\bar{b}$  final state, assuming  $L = 20 \text{ fb}^{-1}$ . The solid (dotted) curves are for  $x_{min} = 0.5(4.0)$  with  $E_{e^+e^-} = E_{e^+e^-}^{opt}$ .

two modes dominate with the  $b\bar{b}$  channel having about 90% branching ratio in the absence of SUSY decay channels.

Non-Higgs backgrounds to these modes are negligible except for the  $b\bar{b}$  and  $t\bar{t}$  channels, for which we include the continuum  $\gamma\gamma \rightarrow b\bar{b}$  and  $\gamma\gamma \rightarrow t\bar{t}$  processes, respectively. As discussed earlier, resolved photon backgrounds are insignificant since the optimal  $\gamma\gamma$  collision energies for measurement of  $\mathcal{A}_3$  are always close to the kinematical limit.

Our results appear in a series of figures. Let us focus first on the  $h^0$ . For  $m_t = 175 \text{ GeV}$  the  $h^0$  has mass below 100(122.5) GeV for  $\tan\beta = 2(20)$  when  $m_{\tilde{t}} = 1 \text{ TeV}$ . Thus, for  $y_h^{opt}(x_{min} = 0.5) = 0.299$  the optimal machine energy always lies below about 410 GeV. Assuming an available (but tunable) energy of  $E_{e^+e^-}^{mach} = 500 \text{ GeV}$ , this means that we can always operate at the optimal point. Only the  $b\bar{b}$  final state is of relevance, for which the values of  $N_{SD}(\mathcal{A}_3)$  appear as the solid lines in Fig. 1. Had we adopted the much less optimal choice of  $x_{min} = 4.0$  the resulting  $N_{SD}(\mathcal{A}_3)$  values would be those indicated by the dotted lines in Fig. 1, *i.e.* about a factor of 5 worse. In contrast, results for  $x_{min} = 1$  are only slightly worse than for  $x_{min} = 0.5$ . From this we conclude that if  $x_{min}$  values of order 0.5 to 1 are possible, there is quite a reasonable chance of being able to determine  $\mathcal{A}_3$  in the case of the

$h^0$ . Only low  $m_{h^0}$  values at low to moderate  $\tan\beta$  values would appear to require integrated luminosities above  $L \sim 20 \text{ fb}^{-1}$ . In contrast, if lasers of adequate power and intensity with energies in the fractional eV range are not possible, much larger integrated luminosity would be required over a broad range of parameter space;  $x_{min} = 4.0$  would require  $L \gtrsim 200 \text{ fb}^{-1}$  in order to measure  $\mathcal{A}_3$  for most  $m_{h^0}$  values.

Figure 2:  $N_{SD}(\mathcal{A}_3)$  for the  $A^0$  is displayed as a function of  $m_{A^0}$  for  $\tan\beta = 2$  and 20 for the  $b\bar{b}$  (solid),  $Zh^0$  (dots) and  $t\bar{t}$  (dashes) final states, assuming  $L = 20 \text{ fb}^{-1}$ , a maximum machine energy of  $E_{e^+e^-}^{mach} = 500 \text{ GeV}$ , and  $x_{min} = 0.5$ .

Turning now to the  $A^0$ , we present results for  $m_{A^0} \geq 40 \text{ GeV}$ . Exactly how high in  $m_{A^0}$  we can go and the range of  $m_{A^0}$  for which we can employ  $x = x_{min} = 0.5$  depends upon the available machine energy,  $E_{e^+e^-}^{mach}$ . We contrast the two cases of  $E_{e^+e^-}^{mach} = 500 \text{ GeV}$  and  $E_{e^+e^-}^{mach} = 1.5 \text{ TeV}$  in Figs. 2 and 3, respectively. At  $E_{e^+e^-}^{mach} = 500 \text{ GeV}$ , one can adjust  $E_{e^+e^-}$  so as to employ  $y_h^{opt}(x_{min} = 0.5)$  so long as  $m_{A^0} \lesssim 150 \text{ GeV}$ , but beyond that  $x$  must be increased above  $x_{min} = 0.5$ ; we have searched for the optimal choice as described earlier. Kinematically, the maximum  $m_{A^0}$  that can be probed at  $E_{e^+e^-}^{mach} = 500 \text{ GeV}$  is just above  $400 \text{ GeV}$  if  $x$  lies below  $4.83$  (*i.e.* below the pair-production threshold). From Fig. 2 we see that at low  $\tan\beta$  the ability to measure  $\mathcal{A}_3$  declines rapidly as soon as we pass the  $t\bar{t}$  threshold at  $m_{A^0} \sim 350 \text{ GeV}$ , but that below that  $N_{SD}(\mathcal{A}_3)$  is significant in the  $Zh^0$  channel.



Figure 3:  $N_{SD}(\mathcal{A}_3)$  for the  $A^0$  is displayed as a function of  $m_{A^0}$  for  $\tan\beta = 2$  and 20 for the  $b\bar{b}$  (solid),  $Zh^0$  (dots) and  $t\bar{t}$  (dashes) final states, assuming  $L = 20 \text{ fb}^{-1}$ , a maximum machine energy of  $E_{e^+e^-}^{mach} = 1.5 \text{ TeV}$ , and  $x_{min} = 0.5$ .

(In drawing this conclusion, we have assumed that this channel is relatively background free in all possible final state modes.) Determination of  $\mathcal{A}_3$  in the  $b\bar{b}$  channel at low  $\tan\beta$  would require enhanced luminosity. At high  $\tan\beta$ , Fig. 2 shows that the  $m_{A^0}$  range over which  $\mathcal{A}_3$  can be measured is considerably diminished. Only the  $b\bar{b}$  final state is useful, and only for  $m_{A^0} \lesssim 70 \text{ GeV}$ . Increased luminosity leads to only a moderate increase in the maximum  $m_{A^0}$  for which  $\mathcal{A}_3$  could be measured; for instance, an accumulated luminosity of  $L \sim 200 \text{ fb}^{-1}$  would extend the  $N_{SD}(\mathcal{A}_3) > 3$  region only to  $m_{A^0} \sim 140 \text{ GeV}$ .

At  $E_{e^+e^-}^{mach} = 1.5 \text{ TeV}$ , increasing  $x$  above  $x_{min} = 0.5$  only becomes necessary for  $m_{A^0} \gtrsim 450 \text{ GeV}$ . Thus, the  $Zh^0$  mode allows even larger  $N_{SD}(\mathcal{A}_3)$  values for  $m_{A^0}$  near  $2m_t$  at low  $\tan\beta$  (see Fig. 3), and even for  $m_{A^0}$  somewhat above  $2m_t$  determination of  $\mathcal{A}_3$  in the  $t\bar{t}$  mode will be possible. The exact reach will depend upon the available integrated luminosity, extending for  $L = 200 \text{ fb}^{-1}$  to possibly as high as  $m_{A^0} \sim 700 \text{ GeV}$ . Unfortunately, prospects for  $\mathcal{A}_3$  determination at high  $\tan\beta$  remain quite limited, despite the increased machine energy. Even for  $L = 200 \text{ fb}^{-1}$  the  $N_{SD}(\mathcal{A}_3) > 3$  region is confined to  $m_{A^0} \lesssim 160 \text{ GeV}$ . It should be noted that increasing  $E_{e^+e^-}^{mach}$  still further does not significantly increase the maximum  $m_{A^0}$  values for which  $\mathcal{A}_3$  can be measured for either low or high  $\tan\beta$ .

Figure 4:  $N_{SD}(\mathcal{A}_3)$  for the  $H^0$  is displayed as a function of  $m_{H^0}$  for  $\tan\beta = 2$  and 20 for the  $b\bar{b}$  (solid),  $ZZ$  (non- $4\nu$ ) (dots),  $h^0h^0$  (dotdash) and  $t\bar{t}$  (dashes) final states, assuming  $L = 20 \text{ fb}^{-1}$ , a maximum machine energy of  $E_{e^+e^-}^{mach} = 1.5 \text{ TeV}$ , and  $x_{min} = 0.5$ .

Finally, let us consider the  $H^0$ . Results for  $N_{SD}(\mathcal{A}_3)$  at  $E_{e^+e^-}^{mach} = 1.5 \text{ TeV}$  and  $x_{min} = 0.5$  are displayed in Fig. 4, assuming  $L = 20 \text{ fb}^{-1}$ . We see that  $\mathcal{A}_3$  will be most observable in the  $h^0h^0$  channel when  $\tan\beta$  is not large and  $m_{H^0} \lesssim 2m_t$ . When  $m_{H^0}$  is very near its lower bound, the  $A^0A^0$  channel also becomes useful (although it is not displayed). For  $m_{H^0} \gtrsim 2m_t$  the  $t\bar{t}$  channel may provide the best opportunity for low  $\tan\beta$ , but  $L$  above  $20 \text{ fb}^{-1}$  will generally be required even for  $x_{min} = 0.5$ . The values of  $N_{SD}(\mathcal{A}_3)$  for the  $ZZ$  channel are given after multiplying rates by  $1 - [BR(Z \rightarrow \nu\bar{\nu})]^2$  (*i.e.* so as to avoid the totally invisible final state mode), but do not include the significant one-loop  $ZZ$  continuum background.<sup>[13]</sup> Inclusion of this continuum background would further diminish the prospects for probing  $\mathcal{A}_3$  in the  $ZZ$  channel. At high  $\tan\beta$ , determination of  $\mathcal{A}_3$  for the  $H^0$  will be very difficult unless the  $H^0$  is very near its lower mass limit (corresponding to small  $m_{A^0}$ ), in which case one of the three channels —  $b\bar{b}$ ,  $h^0h^0$  or  $A^0A^0$  — provides an  $N_{SD}(\mathcal{A}_3)$  value in the range between 5 and 25. (We have not attempted to display this.)

Although not presented, the  $E_{e^+e^-}^{mach} = 500 \text{ GeV}$  figure for the  $H^0$  exhibits differences from Fig. 4 closely analogous to those between Fig. 2 and 3 for the  $A^0$ . The most important

difference in the case of the  $H^0$  is that for  $\tan\beta = 2$  the  $N_{SD}(\mathcal{A}_3)$  value for the  $h^0 h^0$  final state mode declines from the value of  $\sim 5$  shown in Fig. 4 to  $\sim 2.5$  just below the  $t\bar{t}$  threshold.

This concludes our discussion of the basic scenario in which all SUSY partner particles, in particular squarks and inos, are taken to be very heavy, *i.e.* in the 1 TeV mass range. Three variants on this basic scenario have been considered. In the first variant (V1), we have decreased the universal soft squark mass parameter to 350 GeV, maintaining large ino masses by continuing to take  $M = -\mu = 1$  TeV. In the second case (V2), we have kept the soft squark mass large (1 TeV), while allowing small ino masses as determined by  $M = -\mu = 150$  GeV. In the third variant (V3) the soft squark mass is taken to be 350 GeV *and* we have set  $M = -\mu = 150$  GeV. The first case, V1, is interesting in that the radiative corrections to the Higgs masses are greatly reduced, while in V2 the inos are light enough to reduce the SM channel branching fractions for heavier Higgs bosons, while the charginos are light enough to significantly contribute to the one-loop  $\gamma\gamma$  couplings of the different Higgs bosons. Variant V3 combines both effects. We summarize below the effect of the variants V1 and V2 on the achievable  $N_{SD}(\mathcal{A}_3)$  values, assuming a machine energy of  $E_{e^+e^-}^{mach} = 1.5$  TeV. Results for variant V3 are very similar to those for V2 and will not be discussed in detail.

## $h^0$

V1 The maximum  $m_{h^0}$  value declines, but the achievable  $N_{SD}(\mathcal{A}_3)$  values in the  $b\bar{b}$  channel are little affected (at a given  $m_{h^0}$  value). This is true at both low and high  $\tan\beta$ .

V2 The achievable  $N_{SD}(\mathcal{A}_3)$  values decline somewhat (due to the chargino loops cancelling some of the  $W$ -loop contribution to the  $\gamma\gamma$  coupling of the  $h^0$ ), but never by more than about 30%.

## $A^0$

V1 Decreasing the soft squark mass has very little impact upon  $N_{SD}(\mathcal{A}_3)$  at either small or large  $\tan\beta$ . The largest effect is a  $\sim 20\%$  decrease in  $N_{SD}(\mathcal{A}_3)$  in the  $b\bar{b}$  and  $Zh^0$  channels in the  $200 \text{ GeV} \lesssim m_{A^0} \lesssim 2m_t$  range at low  $\tan\beta$ . This is primarily due to small squark-loop contributions to the  $\gamma\gamma$  coupling of the  $A^0$ .

V2 This variant has a major impact at both small and large  $\tan\beta$ . At  $\tan\beta = 2$ ,  $A^0$  decays to ino pairs become significant and one-loop contributions to the  $\gamma\gamma$  coupling of the  $A^0$  are also important. The net effect is to decrease  $N_{SD}(\mathcal{A}_3)$  in all channels, with the maximum result in the  $Zh^0$  ( $t\bar{t}$ ) channel at  $m_{A^0} \lesssim 2m_t$  ( $\gtrsim 2m_t$ ) being reduced to  $N_{SD}(\mathcal{A}_3) \sim 2(3.5)$ , at  $L = 20 \text{ fb}^{-1}$ . Meanwhile,  $N_{SD}(\mathcal{A}_3)$  in the  $b\bar{b}$  channel never rises above 0.4 at  $\tan\beta = 2$ . At  $\tan\beta = 20$ , the coupling of the  $A^0$  to  $b\bar{b}$  is enhanced, and  $BR(A^0 \rightarrow b\bar{b})$  remains large despite the presence of ino-pair channels. Meanwhile, the chargino loops yield a significant increase in the  $\gamma\gamma$  coupling of the  $A^0$ . The result is that  $N_{SD}(\mathcal{A}_3)$  remains above 2 in the  $b\bar{b}$  channel for  $m_{A^0}$  up to  $m_{A^0} \sim 400$  GeV, with a large peak at  $m_{A^0} \sim 220$  where  $N_{SD}(\mathcal{A}_3) \sim 8$ .

## $H^0$

V1 As for the  $A^0$ , decreasing the squark mass to 350 GeV has relatively small impact upon  $N_{SD}(\mathcal{A}_3)$ . At  $\tan\beta = 2$ , the  $N_{SD}(\mathcal{A}_3)$  values achievable in the  $h^0 h^0$  channel for  $m_{H^0} < 2m_t$  decline slightly (by at most 20%), while  $N_{SD}(\mathcal{A}_3)$  increases by up to 50% in the  $t\bar{t}$  channel for  $m_{A^0}$  within 100 GeV of  $2m_t$ . At  $\tan\beta = 20$ , only a tiny decrease in  $N_{SD}(\mathcal{A}_3)$  for the  $b\bar{b}$  channel results from decreasing the squark mass.

V2 The impact of lowering the  $M = -\mu$  value to 150 GeV is once again quite significant. At  $\tan\beta = 2$ ,  $N_{SD}(\mathcal{A}_3)$  in the  $h^0 h^0$  channel falls below 1 for  $m_{H^0} \gtrsim 500$  GeV (as opposed to  $\gtrsim 600$  GeV for our standard scenario). In the  $t\bar{t}$  channel,  $N_{SD}(\mathcal{A}_3)$  only barely reaches 0.9 at its maximum point (as compared to  $\sim 2.2$  in the standard scenario). In contrast, at  $\tan\beta = 20$ ,  $N_{SD}(\mathcal{A}_3)$  is enhanced in the  $b\bar{b}$  channel (just as in the  $A^0$  case), remaining above 1 out to  $m_{H^0} \sim 500$  GeV with a broad peak in the vicinity of  $m_{H^0} \sim 300$  GeV with maximum of  $N_{SD}(\mathcal{A}_3) \sim 2.2$ .

### A Fourth Generation

As a final variant, one might consider the addition of a fourth generation. This can have a dramatic effect. At large  $\tan\beta$ , the observability of  $\mathcal{A}_3$  is dramatically increased for all three neutral Higgs bosons. At small  $\tan\beta$  the impact is more varied. Observability of the  $h^0$  is decreased (due to cancellation of fourth generation loops against the  $W$ -loop contribution to the  $\gamma\gamma$  coupling of the  $h^0$ ). In contrast, observability of the  $A^0$  at small  $\tan\beta$  is dramatically increased since the fourth generation loop contributions to the  $\gamma\gamma$  coupling of the  $A^0$  add to the top-quark loop (and the  $W$  loop is absent). Meanwhile, at small  $\tan\beta$ ,  $N_{SD}(\mathcal{A}_3)$  for the  $H^0$  is not dramatically altered by the addition of a fourth generation.

### **3. Conclusion**

The outlook for measuring the CP-eigenvalues of the three neutral MSSM Higgs bosons via back-scattered photons depends tremendously on the ability to build lasers with high luminosity at low photon energies. At low Higgs mass values, it will be important to be able to adjust the laser energy ( $\omega_0$ ) and electron/positron beam energies so as to be near the optimum values for  $y_h \equiv m_h/E_{e^+e^-}$  while retaining  $x \equiv \frac{2E_{e^+e^-}\omega_0}{m_e^2} \lesssim 1$  in order to have large transverse polarization for the back-scattered photons. For larger values of  $y_h^{mach} \equiv m_h/E_{e^+e^-}^{mach}$  (where  $E_{e^+e^-}^{mach}$  is the maximum available  $e^+e^-$  machine energy), the ability to adjust the laser photon energy to yield  $x$  values not too much larger than the minimum required,  $y_h^{mach}/(1 - y_h^{mach})$ , will again be vital. Of course, we will presumably know in advance the masses of the Higgs bosons, so that in practice only a discrete set of laser energies would have to be available.

Provided fairly optimal choices for the laser and beam energies are possible, prospects for measuring the vital asymmetry  $\mathcal{A}_3$  (which is +1 for a CP-even Higgs boson and -1 for a CP-odd Higgs) are good in the case of the  $h^0$  and are generally good for the  $A^0$  and  $H^0$  at low  $\tan\beta$ , even if only  $20 \text{ fb}^{-1}$  of integrated luminosity is accumulated. At large  $\tan\beta$ , determination of  $\mathcal{A}_3$  for the  $A^0$  and  $H^0$  will generally require significantly larger integrated luminosities (increasing for increasing mass) for all but the lowest mass values. These results (which assume heavy squarks, charginos and neutralinos) are not much altered if squarks

are light. If inos are taken to be light some decline in the statistical significance for a measurement of  $\mathcal{A}_3$  does occur in the case of the  $A^0$  and  $H^0$  at small  $\tan\beta$ , while at large  $\tan\beta$  it generally becomes easier to determine  $\mathcal{A}_3$  for the  $A^0$  and  $H^0$ . Finally, we have noted that the addition of a fourth generation greatly boosts the ease with which  $\mathcal{A}_3$  can be determined for the  $A^0$  and  $H^0$  (but can worsen the prospects for the  $h^0$  if  $\tan\beta$  is small).

While it is likely that the first indication of whether a given neutral Higgs is CP-even or CP-odd will come simply from the size of its  $Z^0 \rightarrow Z^0 h$  production cross section (only a CP-even Higgs has a tree-level coupling of the required type), use of  $\mathcal{A}_3$  appears to provide superior opportunities in comparison to other options for *directly* determining the CP of Higgs bosons.<sup>[14]</sup>

Finally, very high instantaneous photon-photon luminosities appear to be technically feasible,<sup>[15]</sup> by running at correspondingly high instantaneous  $e^+e^-$  luminosities (higher than can be employed without beam disruption *etc.* in direct  $e^+e^-$  collision studies). Although this mode of operation would be expensive using current technology, a combination of money and/or new technology could allow for photon-photon integrated luminosity in the  $\gtrsim 100 \text{ fb}^{-1}$  domain. Measurement of  $\mathcal{A}_3$  would then be even more superior in comparison to other methods for direct determination of the Higgs boson CP-eigenvalues, as referenced above, that rely on the more limited integrated luminosity that would be possible for direct  $e^+e^-$  collision studies.

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